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Sphere drag at Mach numbers from 0·3 to 2·0 at Reynolds numbers approaching 10^7

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Our analysis of 18th and 19th century cannon firings shows that knowledge of sphere drag can be substantially extended into the region of $0·3 \leqslant M_\infty \leqslant 2·0$ and $Re_{\infty d}$ up to 10^7. Bashforth's chronographic measurements (1868) are of a quality comparable to modern measurements. The data of Mayevski (chronograph, 1868), Hutton (ballistic pendulum, 1787–1791), and Didion (ballistic pendulum, 1839–1840) are of lesser accuracy but in agreement with Bashforth's. These cannon data are combined with modern data to provide the most extensive curves available of C_D *vs.* $Re_{\infty d}$ in this region. Interesting features of these curves for $M_\infty \leqslant 1·0$ are briefly described.

1. Introduction

A review of the readily available literature reveals that, of the few studies of sphere drag in the transonic, high Reynolds number flow regime, most have been carried out in aeroballistic ranges (Bailey & Starr 1976; Charters & Thomas 1945; May & Witt 1953; Short 1967; Stilp 1965). In general, the operational characteristics of these ranges limit the maximum operating pressure to approximately 101 kPa (760 torr) and the maximum model size to 50 mm in the transonic speed region. Consequently, the maximum obtainable Reynolds number in ranges is limited to values on the order of 10^6.

Naumann (1954) has made measurements of sphere drag in a wind tunnel at Mach numbers up to 0·9 at Reynolds numbers up to 6×10^5. These results are in good agreement with the above-mentioned aeroballistic range data. Thus, in the transonic speed regime, contemporary measurements of sphere drag are limited to $Re_{\infty d} \leqslant 10^6$ as noted in the six references cited above.

The lack of data in this interesting Mach and Reynolds number regime suggested a consideration of air resistance measurements made when cannonballs were still of major though declining importance in artillery. An examination of old ballistics text-books (Ingalls 1886; Mayevski 1872) yielded sketchy descriptions of several extensive sets of cannon firings for 50 mm (2 in.) to 230 mm (9 in.) diameter spheres in the transonic speed regime (Bashforth 1870, 1881, 1890; Didion 1857, 1860; Hélie 1884; Hutton 1812; Ingalls 1886; Mayevski 1872; Robins 1742). In order to determine the accuracy of drag coefficient values from these experiments, it was necessary to obtain copies of the original detailed reports. This proved to be a difficult task, but such detailed reports of three of these studies have been found (Bashforth 1870, 1881, 1890; Didion 1857, 1860; Hutton 1812). A review of them convinced us that these systematic and carefully controlled studies of the performance of spherical cannon shells could yield useful information concerning sphere drag.

There are a number of reasons why these measurements have been overlooked in comparatively recent studies of sphere drag (Bailey & Starr 1976; Charters & Thomas 1945; May & Witt 1953; Naumann 1954; Short 1967; Stilp 1965). (1) The early measurements were obtained from cannon firings in the 18th and 19th centuries in the course of tests to evaluate the artillery of that time. (2) The experimental results were presented in terms of coefficients that are no longer used. (3) The documents describing these experiments have long been out of print.

The purpose of this paper is (1) to examine these early data, (2) to show that much of the data confirm modern measurements in regions of overlap, (3) to use the best of the data to extend our knowledge of transonic sphere drag into regions of higher Reynolds numbers, and (4) to sketch a little of the historical background.

2. Techniques of early air resistance measurements

The performance characteristics of cannon were determined by measuring the projectile velocity near the muzzle and its subsequent decay with distance from the muzzle. Two techniques were developed to accomplish this.

In early studies of cannon performance (Hutton 1812; Didion 1857), a ballistic pendulum was located at various distances from the gun muzzle, and cannon shells were fired into it. The velocity of the shell at the pendulum could be calculated from a knowledge of the mass of the shell and of the mass, inertia, and velocity of the pendulum. Implicit in this technique was the assumption that the muzzle velocity was repeatable if the same ball weight and powder charge were repeated.

Towards the middle of the 19th century, a number of electrical chronographic timing systems were developed. In this technique, one measured the times at which a cannon shell passed a number of timing stations located at known distances from the muzzle. From such measurements, the variation of velocity with distance from the muzzle could be derived from a single firing (Bashforth 1870, 1881, 1890; Didion 1860; Mayevski 1872).

The main emphasis in the present analysis of these artillery experiments will be placed on Bashforth's studies because they were the most extensive, and his reports contained records of all the physical measurements he made. A lesser evaluation has been made of the other artillery studies primarily because in many cases the original complete data records were not located in the published literature.

3. Experiments made with Bashforth's chronograph (1868)

In 1865, Francis Bashforth, Professor of Applied Mathematics at Woolwich in England, instituted a series of studies with large bore cannons to determine the resistance of the air to the motion of spherical and non-spherical projectiles. Bashforth concluded that the most accurate method for measuring the variation of projectile velocity with distance was to determine the time the projectile took to traverse a series of successive equally spaced, known distances downrange of the gun muzzle. He further concluded that the timing systems available at that time (1865) were not accurate enough for his studies. Therefore, he designed and built an electrical chronograph (figure 1, plate 1). (It is of interest to note that this chronograph is still in existence and can be seen at the Science Museum, South Kensington, London (V. K. Chew, private communication). Bashforth gave the chronograph to the museum in 1876 at the completion of one major phase of his work.)

Bashforth's work was generally ignored by most of the ballisticians of his time, in part because most of it was published in government reports instead of in the more widely distributed artillery journals. There were also those (e.g. Cranz & Becker 1921) who considered that his chronograph was a crude device. On the other hand, Ingalls (1886) in his review of all of the artillery studies available to him at that time stated with regard to Bashforth's work: 'From the data derived from these experiments he constructed and published, from time to time, extensive tables connecting space and velocity, and time and velocity, *which for accuracy and general usefulness have never been excelled.*' Moreover, the fact that Bashforth's original chronograph was used during World War I, 50 years after its design and development, and still 'worked quite satisfactorily' (British Textbook, 1929) suggests that it was capable of accurate timing measurements. Finally, in 1868, a committee was formed to determine (among other things) whether Bashforth's chronograph could measure the small time intervals that he claimed. This committee consisted of Prof. J. C. Adams, Prof. C. G. Stokes, and Captain A. Noble. It was their considered opinion in 1870 that 'we do not think that any means existed before of recording a number of successive small intervals of time with the degree of precision and trustworthiness attained by Prof. Bashforth's instrument' (Bashforth 1870).

Bashforth established what must be considered to be the precursor of present day aeroballistic ranges. He positioned ten detector screens at nine equally spaced distances (45·72 m, 150 ft) with the first one at 22·86 m (75 ft) from the muzzle. The effective range was thus 434 m (1425 ft). The screens were electrically connected together with a current running to the chronograph. When the ball passed through any screen, the current was momentarily interrupted. The chronograph consisted of a paper-covered drum, kept in rotation by a massive flywheel. A pen holder, which moved downward as the drum rotated, contained two pens each connected to a solenoid. One solenoid was connected to the circuit containing the screens; the other, to a circuit containing a switch actuated by the pendulum of an accurate clock. When the clock switch interrupted the current (1 s interval), the clock pen made a tick mark on the otherwise continuous spiral. A similar tick mark was made by the screen pen on its spiral whenever the screen current was interrupted. Thus, the distance between screen tick marks could be related to actual times by direct calibration against the distance between clock tick marks. From the time–distance measurements provided by this system, Bashforth was able to determine the velocity and deceleration of a projectile as a function of downrange distance.

Bashforth fired 198 spherical projectiles of 74 mm (3 in.), 125 mm (5 in.), 176 mm (7 in.), and 225 mm (9 in.) diameter from smooth bore cannons at velocities ranging from 220 to 700 m/s. Information on ambient temperature, ambient pressure, humidity, average projectile weight, average projectile diameter, the time of passage through each of the ten screens, and the average velocity at the mid-point between each of the screens is contained in Bashforth (1870). Moreover, for each of the above diameters he fired hollow and solid spheres to determine the effect of sphere weight on deceleration for a constant diameter projectile. Bashforth did not list the weight and diameter of individual rounds. However, in light of his demonstrated thoroughness in other matters, it is reasonable to assume that round-to-round variations in weight and diameter were so small that he ignored them.

From an analysis of differences of his distance–time measurements, Bashforth concluded that time could be written as a fourth degree polynomial in distance. From this assumption, he was able to determine the velocity and deceleration at any point of the projectile's trajectory. Knowing the deceleration, he was able to calculate the resistance to motion of the projectile, which, in keeping with the theories of that time, he assumed to follow a velocity-cubed law. His resistance coefficient, K_v, was referred to a standard air density, which was 1·2143 and 1·226 kg/m^3 in his 1870 report and 1890 revision, respectively. Furthermore, he recognized that, since K_v was a function of velocity, the cubic law could not be applicable for a wide range of velocities. In the years following 1870, Bashforth conducted a complete review of these firings and concluded that his timing system was more accurate than he had originally thought. Consequently, he published a complete revision of his earlier values of K_v in 1890 based on times now reported to five decimals (Bashforth 1890). It can be shown that:

$$K_v = -\frac{10^9}{144d^2v^3}w\frac{dv}{dt}, \tag{1}$$

where w represents weight (lb or kg) and d diameter. Consequently,

$$C_D = \frac{8}{\pi}\cdot\frac{144K_v v}{10^9\rho_{\text{air}}}, \tag{2}$$

where, to be consistent with Bashforth's English units for K_v, the velocity v must be in ft/s and the density of air ρ_{air} in lb/ft^3. Thus for the 1870 and 1890 tabulations the drag coefficient $C_D = 4{\cdot}838$ and $4{\cdot}805 \times 10^{-6}K_v v$, respectively.

Explicit in Bashforth's analysis is the assumption that K_v is a function of velocity only and is not dependent upon the size of the spherical shell. While such an assumption is valid at supersonic speeds, it is not necessarily true for spheres at subsonic speeds at high Reynolds numbers. Moreover, Bashforth and other ballisticians of the time had quite strong preconceived ideas as to how K_v should vary with velocity. Consequently, there was a tendency to ignore or average in those measurements that did not support their assumptions.

For historical perspective, we note that the classical (19th century) ballistic hypotheses were that the effect of air resistance on a projectile is (1) a function of the projectile's velocity, (2) proportional to the square of its diameter, (3) inversely proportional to its weight, (4) proportional to the air density, and (5) proportional to a shape factor that is independent of the velocity. Indeed Bashforth's experiments for spheres and ogive-cylinder shells (Bashforth 1881, 1890) largely provided the basis for these hypotheses. Although spheres compared to ogive-cylinder projectiles clearly did not have a constant shape factor, hypothesis (5) is nearly true for comparisons among the rather similar ogive-cylinder shapes of 19th century artillery and small arms projectiles. Moreover, Reynolds number effects are somewhat smaller for ogive-cylinder shapes, so that the d^2 hypothesis is quite adequate for them. However, the somewhat larger Reynolds number effect for spheres, which is the focus of our interest, was ignored by Bashforth owing to his preconceived view favouring hypothesis (2).

It was believed that Bashforth's measurements warranted an independent analysis because they could contain information on the drag of spheres at subcritical, critical, and supercritical Reynolds numbers at high subsonic speeds. This flow regime is characterized by a lack of modern experimental data.

Bashforth's original 1870 report contained complete tabulations of (1) time and distance, and (2) velocity and distance for all 198 rounds. The velocities were obtained by dividing the distance between the timing stations by the time taken to traverse this distance. These velocities were tabulated as a function of the mid-point distance between the timing stations. Now the drag experienced by any projectile can be expressed in the following manner:

$$\tfrac{1}{2}\rho_{\text{air}}\, v^2 S C_D = -w\, dv/dt; \tag{3}$$

or

$$\tfrac{1}{2}\rho_{\text{air}}\, v^2 S C_D = -wv\, dv/ds; \tag{4}$$

where S is $\frac{1}{4}\pi d^2$ and s is distance. Therefore,

$$C_D = -\frac{2w}{\rho_{\text{air}}\, v^2 S}\frac{dv}{dt}, \tag{5}$$

$$C_D = -\frac{2w}{\rho_{\text{air}}\, S}\cdot\frac{1}{v}\frac{dv}{ds}. \tag{6}$$

Bashforth obtained values of v and dv/dt by expressing time as a fourth degree polynomial in distance:

$$t = a_1 s + a_2 s^2 + a_3 s^3 + a_4 s^4; \tag{7}$$

hence,

$$v = (a_1 + 2a_2 s + 3a_3 s^2 + 4a_4 s^3)^{-1} \tag{8}$$

and

$$\frac{dv}{dt} = -(2a_2 + 6a_3 s + 12a_4 s^2)\, v^3. \tag{9}$$

In the present analysis, least squares fits have been made of (1) time as a fifth degree polynomial in distance, and (2) mean velocity as a second degree polynomial in distance. The latter is equivalent to t being a third degree polynomial in distance. An example of drag coefficients derived from the velocity–distance relationship is given in figure 2 for the 74 mm (3 in.) diameter hollow shell. The spread in this experimental data is characteristic of that for all shell diameters. A summary of the mean curves fitted to the experimental data is presented in figure 3.

It is apparent from figure 3 that there is little to choose between the above data fits and those derived from Bashforth's K_v values. Therefore, the data obtained from v as a quadratic in s have had a smooth line drawn in by eye for each diameter. Points from these lines have been used to summarize C_D as a function of Reynolds and Mach number in figure 4. Other Reynolds number portions of figure 4 have been constructed from the previously mentioned range and wind tunnel data (Bailey & Hiatt 1971; Bailey & Starr 1976; Charters & Thomas 1945; May & Witt 1953; Naumann 1954; Short 1967; Stilp 1965).

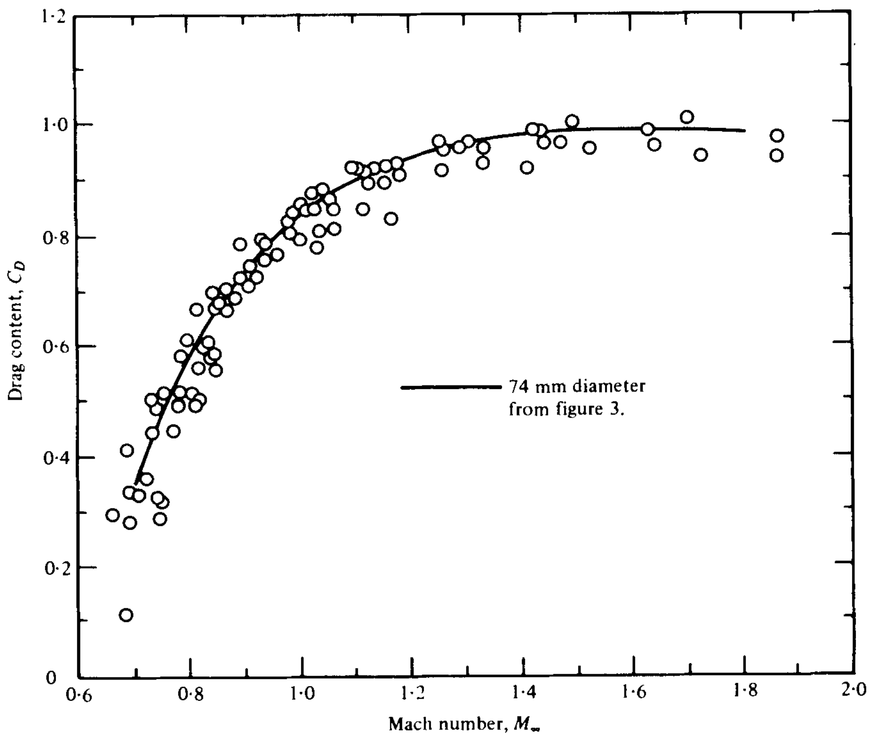

FIGURE 2. Variation of drag with Mach number for 74 mm diameter hollow sphere calculated from Bashforth (1870).

As noted earlier, there is no direct means for determining the accuracy of Bashforth's chronograph. However, there is an indirect method. The results presented by Bailey & Hiatt (1971) indicate that, for $M_\infty \geqslant 1\cdot6$ and $Re_{\infty d} \geqslant 10^5$, sphere drag appears to be constant with further increases in Reynolds number. Because Bashforth's values are in good agreement with the Bailey & Hiatt data in this Mach and Reynolds number regime, it can be assumed that Bashforth's measurements of diameter, weight, temperature, humidity, pressure, distance, and time are of a quality comparable to that which exists for modern measurements in this same flight regime. However, we note that some of Bashforth's rounds are characterized by drag values that vary in an unrealistic manner with Mach number. These rounds, together with those where timing values were obtained for five or fewer stations (43 altogether), were not considered in the present analysis. Although C_D is shown in figure 4 to be invariant with increase in Reynolds number for $R_{\infty d} > 10^6$ and $M_\infty > 0\cdot9$, it should be noted that the 225 mm data indicate possible decrease in C_D with increasing $Re_{\infty d}$.

Sphere drag measurements obtained by Naumann (1954) at high subsonic Mach numbers in a wind tunnel are in good agreement with figure 4 and with the results obtained in free flight. Of particular interest is the good agreement between these two sets of measurements at $M_\infty = 0\cdot7$ where, at approximately the same Reynolds number, they both show a rapid decrease in drag with increasing Reynolds number. At lower Mach numbers, a decrease in drag of this type has been associated with a change in the flow over the sphere from laminar to turbulent.

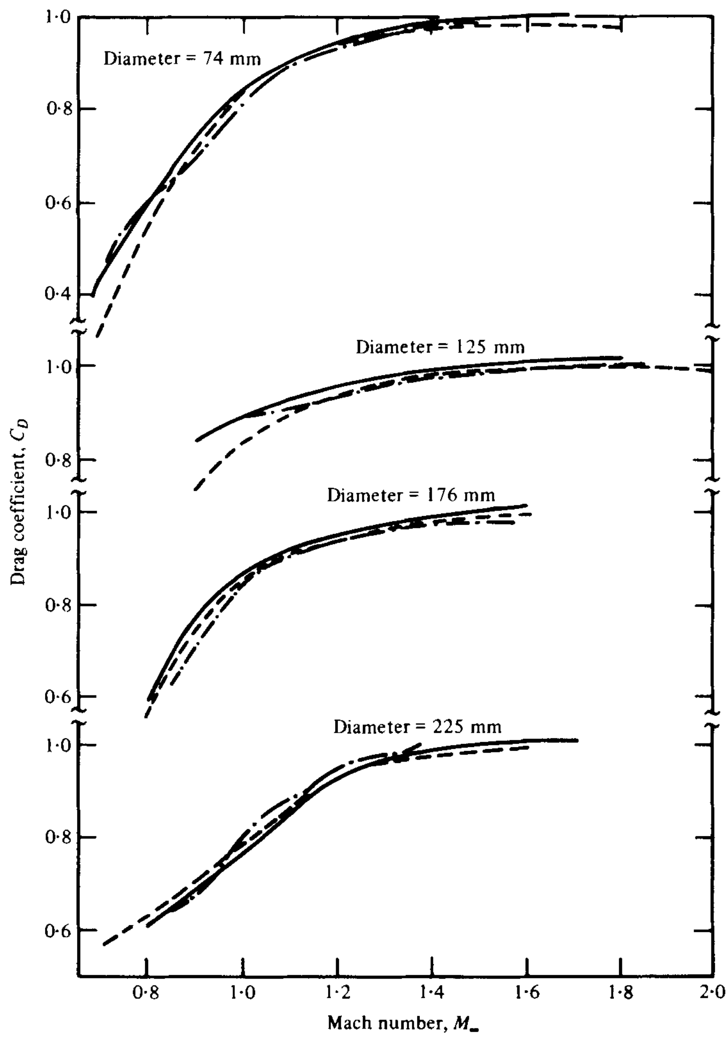

FIGURE 3. A comparison of data reduction methods for Bashforth's sphere measurements (——, derived from a 5th degree fit to tabulated values of distance and time $t = a_1 s + \ldots + a_5 s^5$; — · — ·, derived from Bashforth's tabulated K_v values based on $t = a_1 s + \ldots a_4 s^4$; – – –, derived from a 2nd degree fit to tabulated values of velocity and time – equivalent to $t = a_1 s + \ldots + a_3 s^3$).

4. Metz chronograph experiments (1856–1858)

The results of a French investigation carried out in 1856, 1857, and 1858 at Metz using an electro-ballistic pendulum designed by Captain Navez of the Belgian Artillery are described briefly by Ingalls (1886), Didion (1857, 1860) and Mayevski (1872). With this device, it was possible to measure the velocity at two points on the trajectory of each round. A 22 cm howitzer and 8 and 24 lb cannon were used to launch spherical projectiles having diameters of 220, 100, and 148 mm, respectively, at velocities ranging from 190 to 550 m/s. From a more detailed summary, presented by Hélie (1884) of the ten 148 mm projectile firings, it appears that between 8 and 30 shots were fired for each ball weight and powder charge. Three values of C_D calculated

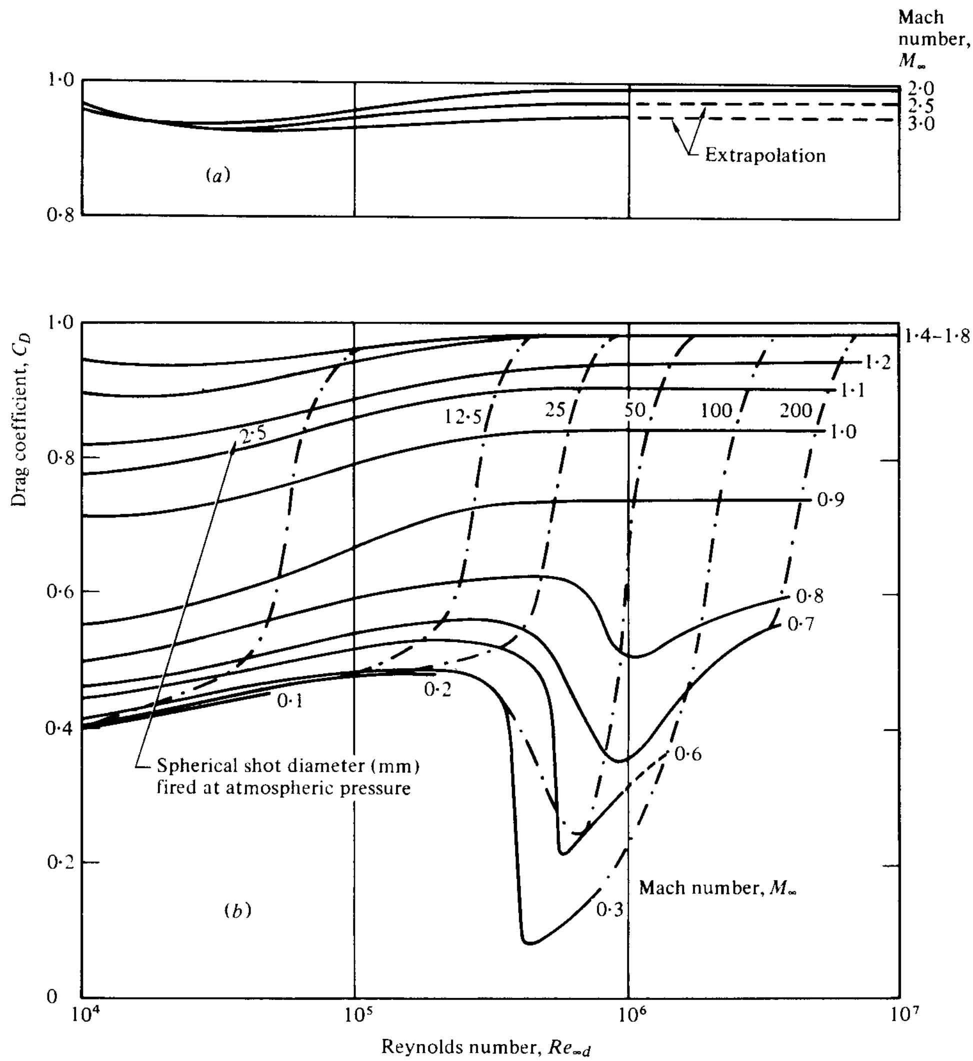

FIGURE 4. Summary of sphere drag measurements at high Reynolds numbers: (a) $2{\cdot}0 \leqslant M_\infty \leqslant 3{\cdot}0$, and (b) $0{\cdot}2 \leqslant M_\infty \leqslant 1{\cdot}8$ (curve is in two parts because C_D reaches a maximum between $M_\infty = 1{\cdot}6$ and $1{\cdot}8$).

directly from Hélie's tabulations of velocity as a function of distance are not consistent with C_D calculated from Hélie's calculated drag function. Moreover, these 148 mm data are largely inconsistent with C_D calculated from the drag function tabulated in Mayevski's summary for this size. Values of C_D for the 148 mm diameter projectile calculated from Hélie's velocity–distance tabulation are given in figure 5, along with the data for the 100 and 200 mm diameter projectiles calculated from Mayevski's summary.

Because most of the results obtained using the Navez device are in poor agreement with Bashforth's measurements for $1{\cdot}1 \leqslant M \leqslant 1{\cdot}6$, it has been concluded that this electro-ballistic pendulum was not accurate enough for cannon performance studies at high speeds. However, the electroballistic pendulum may give reasonable values for $M_\infty < 1{\cdot}1$.

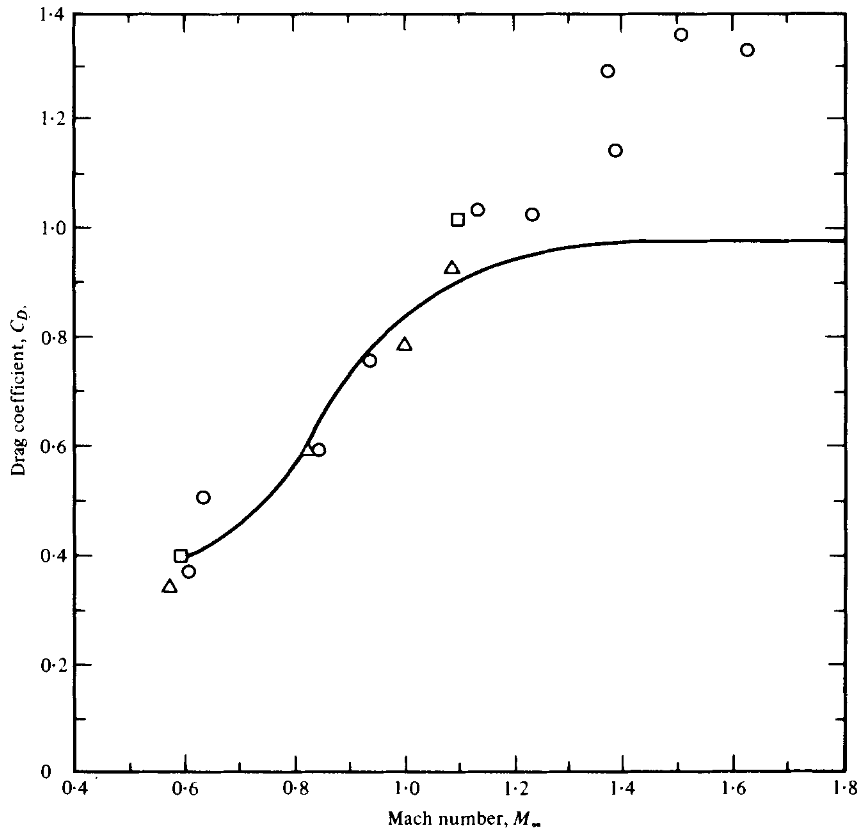

FIGURE 5. Metz sphere drag measurements obtained with the Navez chronograph, 1857–1859. Diameters (mm) are: ○, 148 (Hélie 1884); △, 220 (Mayevski 1872); □, 103 (Mayevski 1872); ——, 148 (from figure 3).

5. Mayevski experiments (1868)

The Russians suspected problems with the 1856–1858 Metz experiments. Consequently, Mayevski (1872) conducted a series of tests at St Petersburg in 1868 with spherical projectiles ranging in size from 91 to 244 mm at speeds ranging from 227 to 527 m/s. He used two Boulengé chronographs to measure the velocity, one at each of two points on the trajectory separated by a known distance. Mayevski fired at least eight shots for each ball weight and powder charge, and his final results are presented as averages of the firings. Because individual firing data are not available, it has not been possible to determine the degree of scatter of the measurements. Mayevski's air resistance coefficients have been converted to C_D and are presented as a function of Mach number in figure 6. Also shown in this figure are the values derived from figure 4. The good measure of agreement between these data and figure 4 indicates that the Boulengé chronograph is capable of making accurate measurements of velocity and, hence, sphere drag coefficients.

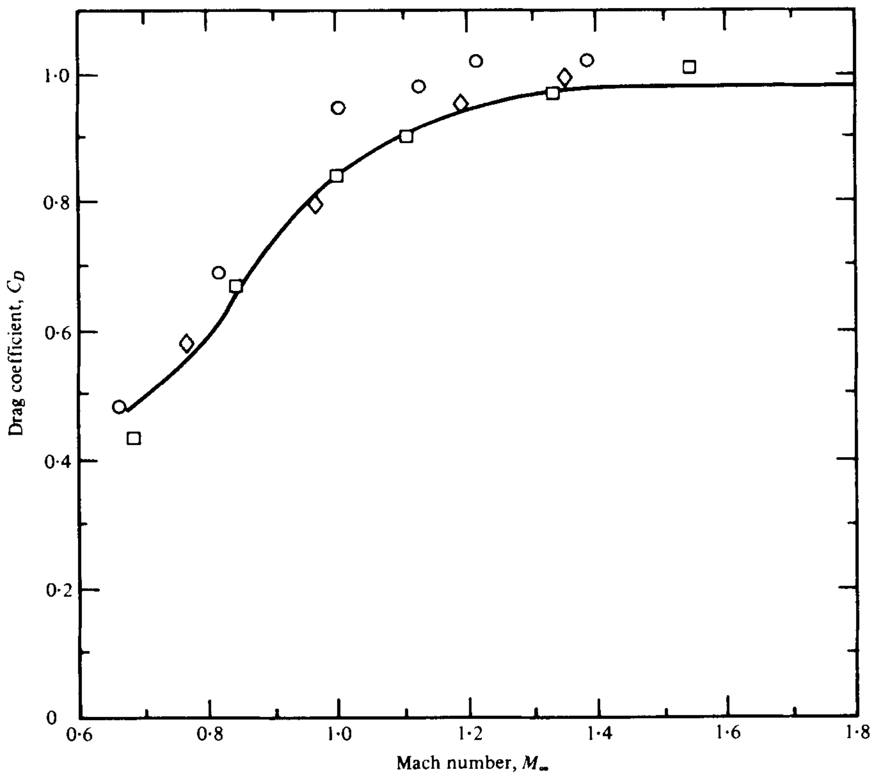

FIGURE 6. Mayevski sphere drag measurements obtained with the Boulengé chronograph, 1868 (Mayevski 1872). Diameters (mm) are: ○, 91; □, 148; ◇, 244; ——, 148 (from figure 3).

6. Experiments made with the ballistic pendulum

6.1. *Robins*

Robins' invention of the ballistic pendulum led to the first air resistance measurements during 1740–1742 (Robins 1742). His results for 19 mm ($\frac{3}{4}$ in.) musket balls weighing about 28 g showed the characteristic increase in drag in the velocity of sound region, and proved that the Newtonian v^2 law was not correct near and above sonic velocity. These pioneering results are too inaccurate for our use.

6.2. *Hutton*

Hutton (1812) improved the design of the pendulum, and during 1775–1791 carried out cannon firings using balls of 50 mm (2 in.), 71 mm (2·8 in.), and 90 mm (3·5 in.) weighing 450–2750 g with muzzle velocities from 91 to 630 m/s. In this paper, we concern ourselves with his ballistic pendulum experiments carried out in the years 1787–1791. His reports (Hutton 1812) are very detailed and contain sufficient information to determine the variation of projectile velocity with distance from the muzzle. This variation together with a knowledge of the projectile weight, projectile diameter, ambient temperature, and ambient pressure make it possible to calculate the variation of drag coefficient with distance.

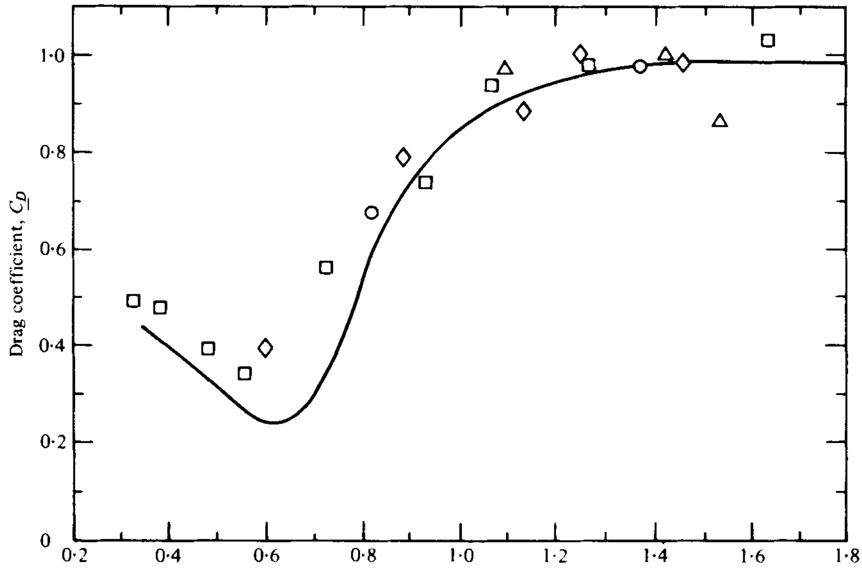

FIGURE 7. Sphere drag measurements obtained with Hutton's ballistic pendulum (Hutton 1812). Diameters (mm) are: ○, 50 (1787); □, 50 (1788); ◇, 71 (1789); △, 90 (1791); ——, 50 (from figure 3).

Cannon performance in Hutton's time was characterized by the poor repeatability of muzzle velocity for the same powder charge and ball weight. After much experimentation, he found that a major reason for this poor repeatability was the poor reproducibility of the black powder. He devised a means of producing black powder charges of repeatable quality, resulting in a significant reduction in the spread of the muzzle velocity from a given powder charge and ball weight.

Hutton's procedure was to fire the same charge weight and ball weight, but with the pendulum successively at several distances increasing from 9·14 m (30 ft) to 109·73 m (360 ft). At each distance, several repeat shots were fired to average out the uncertainty in the pendulum measurements and in the muzzle velocity. The results gave velocity as a function of distance for an unmeasurable but approximately constant muzzle velocity. Then the powder charge was changed, and the procedure repeated. In this way, velocities were measured between 91 and 630 m/s (300 and 2000 ft/s).

In the present analysis, the average velocity was plotted as a function of distance from the gun muzzle for each powder charge. A linear fit was made to this velocity–distance variation, and the drag coefficient determined using equation (6). The velocity drop for a number of the test conditions is so large that it is not correct to assume a linear variation of velocity with distance. However, it was believed that the quality of the average velocity measurements was not sufficiently good to warrant fitting them to a higher-order polynomial.

The results of the present linear analysis are shown in figure 7. It can be seen that for $0{\cdot}8 \leqslant M_\infty \leqslant 1{\cdot}6$, these results are characterized by a comparatively small degree of scatter and are in reasonable agreement with figure 4, which is based primarily on Bashforth's values. For lower speeds ($M_\infty \leqslant 0{\cdot}8$), the variation of C_D with M_∞ is in only fair agreement with figure 4, but is qualitatively similar. Moreover, this variation of C_D with M_∞ is characteristic of that which would be expected as the flow over the sphere changes from laminar to turbulent.

Didion (1857) made an extensive analysis of Hutton's work. As a result of the studies at Metz, 1839–1840 (Didion 1857) described below, Didion became concerned with the effects of (1) the muzzle gas impinging on the pendulum, and (2) the curvature of the projectile's trajectory due to the effect of gravity. Didion recalculated Hutton's work taking these two factors into consideration and tabulated these values (Didion 1857). Didion's recalculated resistance coefficient values have been transformed into C_D values. For $M_\infty \geqslant 1{\cdot}0$ his values are in reasonable agreement with ours, whereas for $M_\infty < 1{\cdot}0$ his values are somewhat higher than the results of the present analysis. The corrections for muzzle gas effects and trajectory curvature are not large enough to account for these differences. The present analysis would indicate that the corrections used by Didion were not really warranted by the quality of the basic data. Our analysis does have an element of subjectivity in determining best how to fit a straight line to the velocity–distance values. However, the uncertainty in making this fit is not large enough to produce values of C_D comparable to those derived from the Didion analysis (Didion 1857).

6.3. *Didion*

Didion (1857) also reported on an extensive ballistic pendulum study at Metz (1839–1840) on the performance of 103, 118, 148, and 220 mm diameter spherical shells for a range of powder charges. This study was not as well designed as Hutton's, because for most of these experiments the velocity was measured at only two locations downrange. Many shots were fired for each powder charge and ball diameter, so that a reasonably accurate average velocity measurement was obtained at each pendulum position. Drag coefficients calculated from a knowledge of the velocity drop between the pendulum stations using equation (6) are shown in figure 8. Twelve of the fifteen data points shown are in good agreement with the values derived from Bashforth's data (figure 4). It can be shown that the present results, which do not contain corrections for muzzle gases and trajectory curvature, are in good agreement with Didion's values, which do (Didion 1857). This suggests that Didion's corrections are of limited importance for these experiments as well as for Hutton's.

7. Conclusions

It is apparent from the preceding discussion that, regardless of the technique used to determine cannon performance, the resulting values of sphere drag coefficient are characterized by an uncertainty of at least ± 5 percent. Contemporary measurements of sphere drag coefficient for spheres ranging in size from 30 to 50 mm are summarized in figure 9. It is readily apparent that these measurements are also characterized by a comparable degree of scatter.

It has been shown that the sphere drag values derived from Bashforth's extensive cannon firings are self-consistent and are consistent with comparable measurements made at lower Reynolds number in aeroballistic ranges. However, modern sphere drag measurements in the Mach number range 0·3–2·0 have been limited to $Re_{\infty d} \leqslant 10^6$. The present study shows that Bashforth's experiments can extend this Reynolds

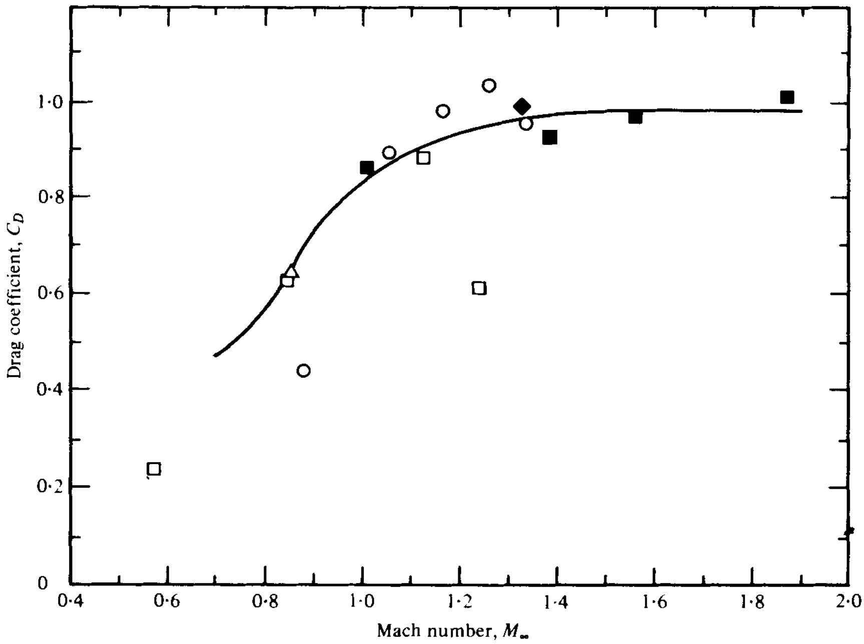

FIGURE 8. Sphere drag obtained with Hutton's ballistic pendulum, 1839–1840 (Didion 1857). Diameters (mm) are: ○, 148; □, 181; ◇, 103; △, 220; ——, 148 (from figure 3). Open symbols: C_D based on two velocity measurements; solid symbols: C_D based on more than two velocity measurements.

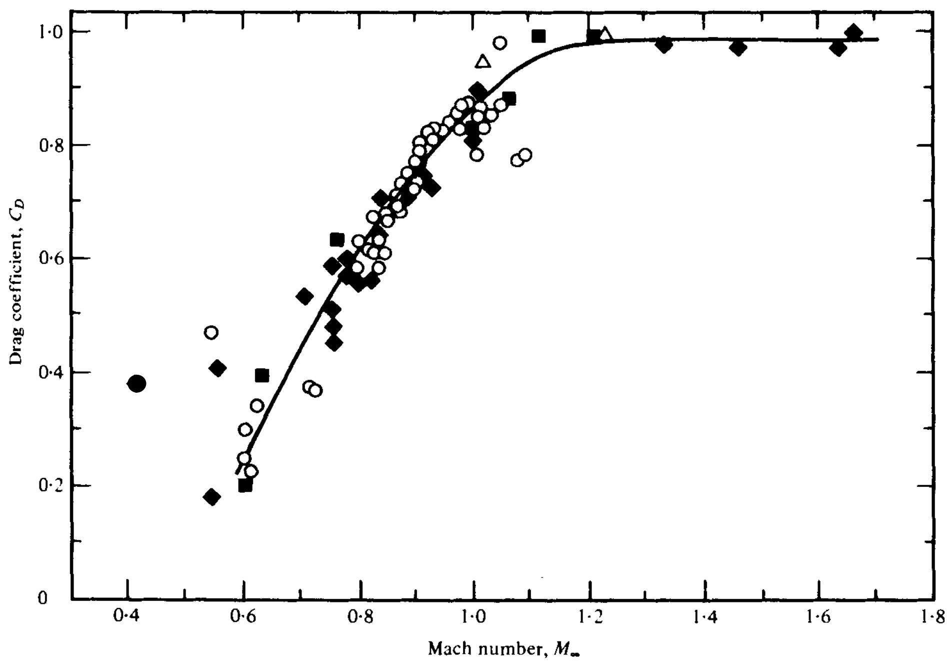

FIGURE 9. Summary of modern 30–50 mm diameter ballistic range data. Diameters (mm) are: ○, 30–50 (Stilp 1965); ■, 38 (Charters & Thomas 1945); ◆, 41 (Short 1967); △, 38 (Bailey & Starr 1976).

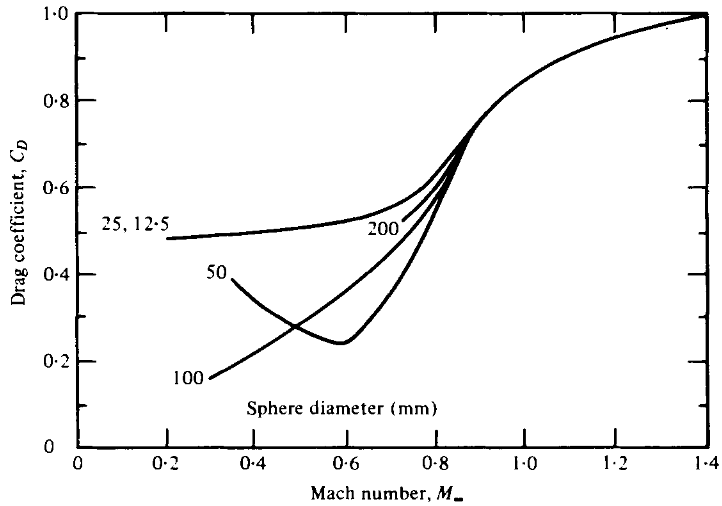

FIGURE 10. Variation of drag coefficient with Mach number for various diameters at atmospheric pressure.

number range to approximately 10^7. This study also provides unique information on the drag of a sphere at high subsonic Mach numbers when the flow over the sphere is turbulent. Moreover, an analysis of all the readily available cannon data obtained in the 18th and 19th centuries has shown that sphere drag coefficients derived from ballistic pendulum techniques or chronographic timing systems, with the exception of the Navez results obtained at Metz (1858–1859), are in agreement with the more extensive results obtained with Bashforth's chronograph.

Results of our analysis of Bashforth's cannon firings together with modern data have been used to construct figure 4. To our knowledge, this is the most complete summary of C_D as a function of $Re_{\infty d}$ for Mach numbers ranging from 0·1 to 3·0. Values of C_D for $2 \leqslant M_\infty \leqslant 3$ have been plotted on a separate insert in figure 4 to avoid cluttering the graph.

As indicated in earlier work and confirmed here, for $M_\infty \leqslant 1$, figure 4 shows a marked dip in $Re_{\infty d}$ ranges, which correspond to 25–30 mm (2–4 in.) diameter spheres at atmospheric pressure. Note that this dip gets smaller and levels out as sonic velocity is approached.

Figure 10 shows C_D *vs.* M_∞ taken from figure 4 to correspond to drag at atmospheric pressure for various diameter spheres. Between 12·5 and 25 mm diameter, C_D is essentially independent of size. The 200 mm diameter curve is about the same as the one for 12·5–25 mm down to the lowest M_∞ available. However, at intermediate diameters, there are very striking differences, as much as a factor of 2 in C_D from $0{\cdot}2 \leqslant M_\infty \leqslant 0{\cdot}8$. Although not shown in figure 10, the curve for $d = 2{\cdot}5$ mm is systematically lower than for the 12·5–25 mm curve by about 0·06–0·1 in C_D. Such Reynolds number effects are very much less for ogive-cylinder or cone-cylinder projectiles. The substantially lower drag of intermediate-sized spheres implies, in the terms of a 19th century artilleryman, that such cannonballs (2–4 in.) carry farther than expected compared to musket balls (0·5–0·75 in.) even when the distance is scaled 'properly' by the 19th century ballistic scaling factor w/d^2.

Note that data are still lacking in the important region where the curves in figure 4 are changing rapidly or in figure 10 show such large differences; namely, where $M_\infty < 1$ and the diameter is 25–225 mm (1–9 in.). However, the operational characteristics of existing wind tunnel and ballistic range facilities are such that sphere drag measurements cannot be made at the high Reynolds numbers characteristic of these mid-19th century cannon firings. Should the need arise to confirm these cannon firings or to extend the results into the interesting region noted above, a free air range of the type established by Bashforth with modern timing, surveying, and detecting techniques would be appropriate.

We are indebted to Sandy Love and Mary Allen of the L.L.L. Library for their perseverance in obtaining the rare 18th and 19th century documents. We would also like to credit the genius and ability of those 18th and 19th century ballisticians, such as Bashforth, Mayevski, Didion and Hutton. Their reputations may have faded with time and circumstance, but their experimental work has not.

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